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Nonlinear Behavior of Sandwich Panels with a Transversely Flexible Core

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Nonlinear behavior of sandwich panels with a transversely flexible core is presented. The study is based on a high-order nonlinear formulation that includes the influence of the transverse flexibility and shear resistance of the core on the panel behavior, thus allowing for interaction between the facings through the core thickness. The solutions obtained are general and are not based on decoupling of the local and global responses as commonly used in the literature. The governing equations along with the appropriate boundary and continuity conditions are presented, and the solution approach is outlined. The path-following algorithm devised is based on the natural parameter and the arc-length continuation techniques. The example problems discussed include a concentrated line load exerted at midspan of the panel, couples applied at the panel edges, and compression of an asymmetric sandwich panel. Mode interaction observed in the panel behavior is shown to be a result of the flexibility of the "soft" core. Variations in the boundary conditions of the sandwich panel as well as in the layout of the compressive longitudinal loads are also shown to shift its response from an imperfection-sensitive to an imperfection-nonsensitive one and vice versa.

I. Introduction

IN recent years sandwich constructions with transversely flexible ("soft") cores have received wide acceptance from various fields of industry. A demand for the analytical tool capable of adequate description for such structures has been met by works of Frostig et al.[1] and Frostig and Baruch.[2] They proposed a refined closed-form high-order sandwich panel theory (HSAPT) that allows for analyzing the general sandwich panel layout with a transversely flexible core subjected to various types of external loading. The main advantages of the HSAPT are that different boundary conditions can be imposed on the various face sheets at the same section allowing the actual supporting cases to be analyzed, the nonlinear patterns of stresses and deformations through the thickness of the core are the result of the theory and not a priori assumed, the general solution is obtained, i.e., the analysis does not separate the complicated sandwich response into isolated problems corresponding to the global and local modes used by many researchers in the field of sandwich structures (see, for example, Refs. 3–5). The important simplifying assumption invoked by the HSAPT consists in neglecting the longitudinal rigidity of the core. This assumption is used whenever an ordinary sandwich structure is of concern,[3,6] and it is valid for structures with cores made of honeycombs, typical foams, or low-strength honeycombs. The recent work by Thomsen and Frostig[7] has presented the experimental verification of the HSAPT with respect to the stress field induced by the application of a strongly localized external loading or point supports.

Sokolinsky and Frostig[8] applied the HSAPT to buckling analysis of arbitrarily supported sandwich panels with the soft core subjected to nonmembrane regime in the prebuckling state. The results obtained confirmed the tendency of the lightweight structures for more complex forms of buckling response as was indicated by Hunt et al.[4] However, at the same time the application of the HSAPT made it possible to obtain generally complicated buckling responses for which the interaction between the overall (global) mode and the local or localized modes takes place. The local response is defined as a deformation pattern that is located at the face sheets only such as wrinkling, whereas the localized response is interpreted as a deformation pattern that is confined to some zone along the panel.

The present work uses the HSAPT to analyze a geometrically nonlinear general behavior of sandwich panels with the soft core within the limits of intermediate class of deformations,[9] i.e., large deformations with moderate rotations and small strains.

II. Nonlinear Equilibrium Equations

The governing nonlinear differential equations for the sandwich panel with the soft core (see Figs. 1 and 2), their associated boundary and continuity conditions, as well as the core fields along with the assumptions used appear in Frostig and Baruch[2] and are presented here for completeness.

The assumptions used by the HSAPT are as follows (for more details see Refs. 1 and 2): the sandwich panel behaves in the linear elastic range; the face sheets are modeled by ordinary panels (with negligible shear) that follow Euler–Bernoulli assumptions and are subjected to the intermediate class of deformations[9] and the core layer is considered as a two-dimensional elastic medium with small deformations where the height of the soft core may change under loading, whereas its cross-section plane does not remain plane; see Fig. 2b. The longitudinal (in-plane) stresses are neglected, similar to the assumption used for an antiplane core.[3] Neglect of the longitudinal rigidity of the flexible core made of honeycombs or typical foams is fully justified because its modulus of elasticity and its flexural rigidity are about three and two orders smaller than their counterparts for the face sheets, respectively.[1] The interface layers are assumed to resist shear and vertical normal (peeling) stresses and to provide a full bond between the core and face sheets, i.e., compatibility of the displacements exists in the vertical and the longitudinal directions at the face sheets–core interfaces. The term *panel* here applies either to a narrow beam or a wide beam (a plate in cylindrical bending). However, in case of the face sheets made of an isotropic material the cylindrical bending of wide beams is accounted for through multiplying the Young's modulus of each face sheet by $1/(1-\nu^2)$, where ν is the Poisson's ratio of the face sheet.[3]

The field equations have been derived through the variational principle of minimum potential energy, Euler–Bernoulli kinematic relations for the face sheets, and using compatibility conditions of full bond at face sheets–core interfaces. Hence, they read

$$N_{xxt,x} + b\tau = -n_t \tag{1}$$

$$N_{xxb,x} - b\tau = -n_b \tag{2}$$

$$M_{xxt,xx} + (N_{xxt}w_{t,x})_{,x} + \frac{bE_c}{c}(w_b - w_t) + \frac{b(c+d_t)}{2}\tau_{,x} = -q_t \tag{3}$$

$$M_{xxb,xx} + (N_{xxb}w_{b,x})_{,x} - \frac{bE_c}{c}(w_b - w_t) + \frac{b(c+d_b)}{2}\tau_{,x} = -q_b \tag{4}$$

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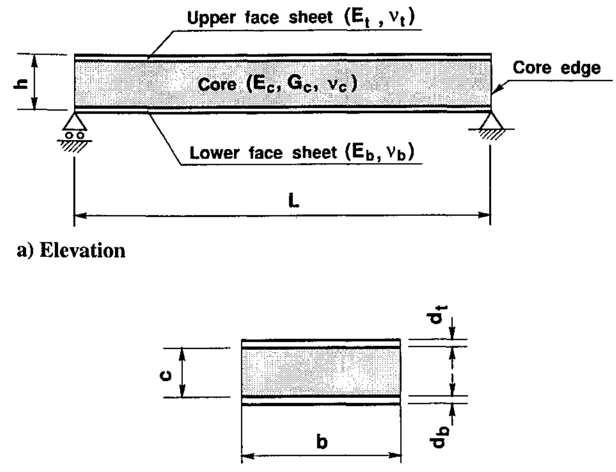
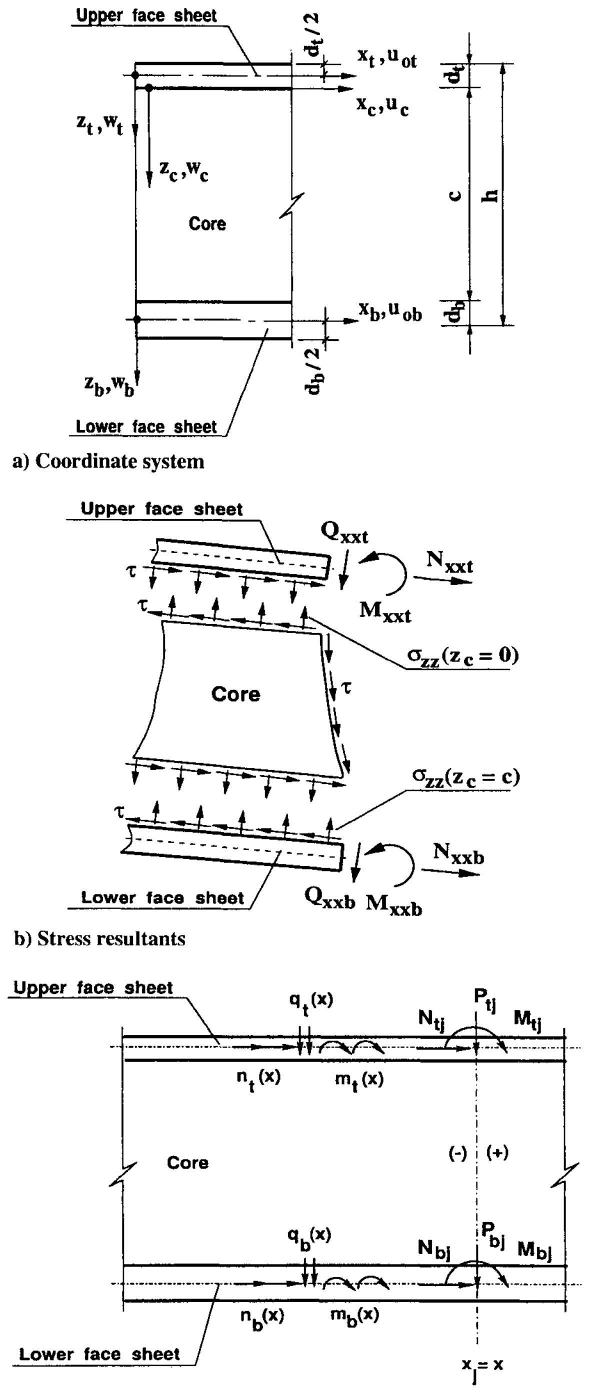


a) Elevation

b) Cross section

Fig. 1 Sandwich panel layout.



a) Coordinate system

b) Stress resultants

c) External loads

Fig. 2 Sandwich panel notation.

$$u_{ot} - u_{ob} - \frac{c + d_t}{2} w_{t,x} - \frac{c + d_b}{2} w_{b,x} - \frac{c^3}{12E_c} \tau_{,xx} + \frac{c}{G_c} \tau = 0 \tag{5}$$

where the unknowns u_{ot}, u_{ob}, w_t, w_b, and τ are the horizontal and the vertical displacements of the centroid line of the upper and the lower face sheets and the shear stress in the core, respectively (Figs. 2a and 2b); $()_{,x}$ denotes the derivative with respect to x, whereby x is implied in any of three axes x_t, x_b, or x_c ($x_t = x_b = x_c = x$); b, c, d_t, and d_b denote the width of the panel cross section, the height of the core, and the thickness of the upper and the lower face sheets, respectively (Figs. 1b and 2a); G_c, E_c are the shear and the elastic moduli of the core; n_t, n_b are external horizontal and q_t, q_b external vertical distributed forces applied to the upper and the lower face sheets, respectively (Fig. 2c); N_{xxt}, N_{xxb} are force stress resultants and M_{xxt}, M_{xxb} are moment stress resultants in the upper and the lower face sheets, respectively (Fig. 2b). Denoting, for brevity, the face sheets by the index i ($i = t, b$), the stress resultants in terms of the normal stresses σ_{xx} (Fig. 2b) are defined as

$$N_{xxi} = b \int_{-d_i/2}^{d_i/2} \sigma_{xxi} \, \mathrm{d}z_i, \qquad M_{xxi} = b \int_{-d_i/2}^{d_i/2} \sigma_{xxi} z_i \, \mathrm{d}z_i \tag{6}$$

In the case of isotropic face sheets, their constitutive relations read ($i = t, b$)

$$N_{xxi} = EA_i \left(u_{oi,x} + w_{i,x}^2 / 2 \right), \qquad M_{xxi} = -EI_i w_{i,xx} \tag{7}$$

where EA_i and EI_i are the longitudinal and flexural rigidities of the face sheets, respectively.

The boundary conditions for the upper and the lower facings at the left ($x = 0$) and the right ($x = L$) edges are given next.

At the face sheets ($i = t, b$):

$$\alpha N_{xxi} = N_i \quad \text{or} \quad u_{oi} = \bar{u}_{oi} \tag{8}$$

$$-\alpha M_{xxi} = M_i \quad \text{or} \quad w_{i,x} = \bar{w}_{i,x} \tag{9}$$

$$\alpha [M_{xxi,x} + N_{xxi} w_{i,x} + (bd_i/2)\tau] = P_i \quad \text{or} \quad w_i = \bar{w}_i \tag{10}$$

At the core:

$$\tau = 0 \quad \text{or} \quad w_c(x, z_c) = \bar{w}_c(z_c) \tag{11}$$

where L is the length of the panel (Fig. 1a); $\alpha = -1$ for $x = 0$ and $\alpha = 1$ for $x = L$; N_i, P_i, and M_i are the horizontal and vertical external loads and bending moments, respectively, applied at the edges of the face sheets; w_c is the unknown function of two variables x_c and z_c, denoting the vertical displacement of an arbitrary point within the core; $\bar{u}_{oi}$, $\bar{w}_i$, and $\bar{w}_{i,x}$ are the specified values of the longitudinal and vertical displacements and the angles of rotation at the boundaries of the face sheets, respectively; $\bar{w}_c(z_c)$ are the specified vertical displacements through the thickness of the core at its edges (Fig. 1a).

The continuity conditions at $x = x_j$ (Fig. 2c), are as follows.

At the face sheets ($i = t, b$):

$$u_{oi}^{(-)} = u_{oi}^{(+)} \tag{12}$$

$$w_i^{(-)} = w_i^{(+)} \tag{13}$$

$$w_{i,x}^{(-)} = w_{i,x}^{(+)} \tag{14}$$

$$N_{xxi}^{(-)} - N_{xxi}^{(+)} = N_{ij} \tag{15}$$

$$-M_{xxi}^{(-)} + M_{xxi}^{(+)} = M_{ij} \tag{16}$$

$$M^{(-)}_{xxi,x} + N^{(-)}_{xxi} w^{(-)}_{i,x} + \frac{b^{(-)} d_i^{(-)}}{2}\tau^{(-)} - M^{(+)}_{xxi,x} - N^{(+)}_{xxi} w^{(+)}_{i,x} - \frac{b^{(+)} d_i^{(+)}}{2}\tau^{(+)} = P_{ij} \tag{17}$$

At the core:

$$\tau^{(-)} = \tau^{(+)} \tag{18}$$

$$w_c^{(-)}(x_c = x_j, z_c) = w_c^{(+)}(x_c = x_j, z_c) \tag{19}$$

where the signs $(-)$ and $(+)$ are left and right of $x = x_j$; N_{ij}, P_{ij}, and M_{ij} are external concentrated loads in the longitudinal and the vertical directions and a concentrated moment exerted at the upper and the lower face sheets, respectively, at $x = x_j$ (see Fig. 2c).

The vertical stress fields as well as the vertical and horizontal displacements of the core, respectively,[2] read

$$\sigma_{zz}(x_c, z_c) = (E_c/c)(w_b - w_t) + (c/2 - z_c)\tau_{,x} \tag{20}$$

$$u_c(x_c, z_c) = u_{ot} + \left(\frac{z_c^2}{2c} - z_c - \frac{d_t}{2}\right) w_{t,x} - \frac{z_c^2}{2c} w_{b,x} - \frac{z_c^2(3c - 2z_c)}{12E_c}\tau_{,xx} + \frac{z_c}{G_c}\tau \tag{21}$$

$$w_c(x_c, z_c) = \left(1 - \frac{z_c}{c}\right) w_t + \frac{z_c}{c} w_b - \frac{z_c(z_c - c)}{2E_c}\tau_{,x} \tag{22}$$

Notice that these stress and deformation fields are the result of a closed-form solution of the partial differential equations of equilibrium of the transversely flexible core determined in Ref. 2 and given next:

$$\tau_{,x} + \sigma_{zz,z_c} = 0 \tag{23}$$

$$\tau_{,z_c} = 0 \tag{24}$$

For more details on the solution of this system of equations, see Ref. 2. Note that the existence of the closed-form solutions in terms

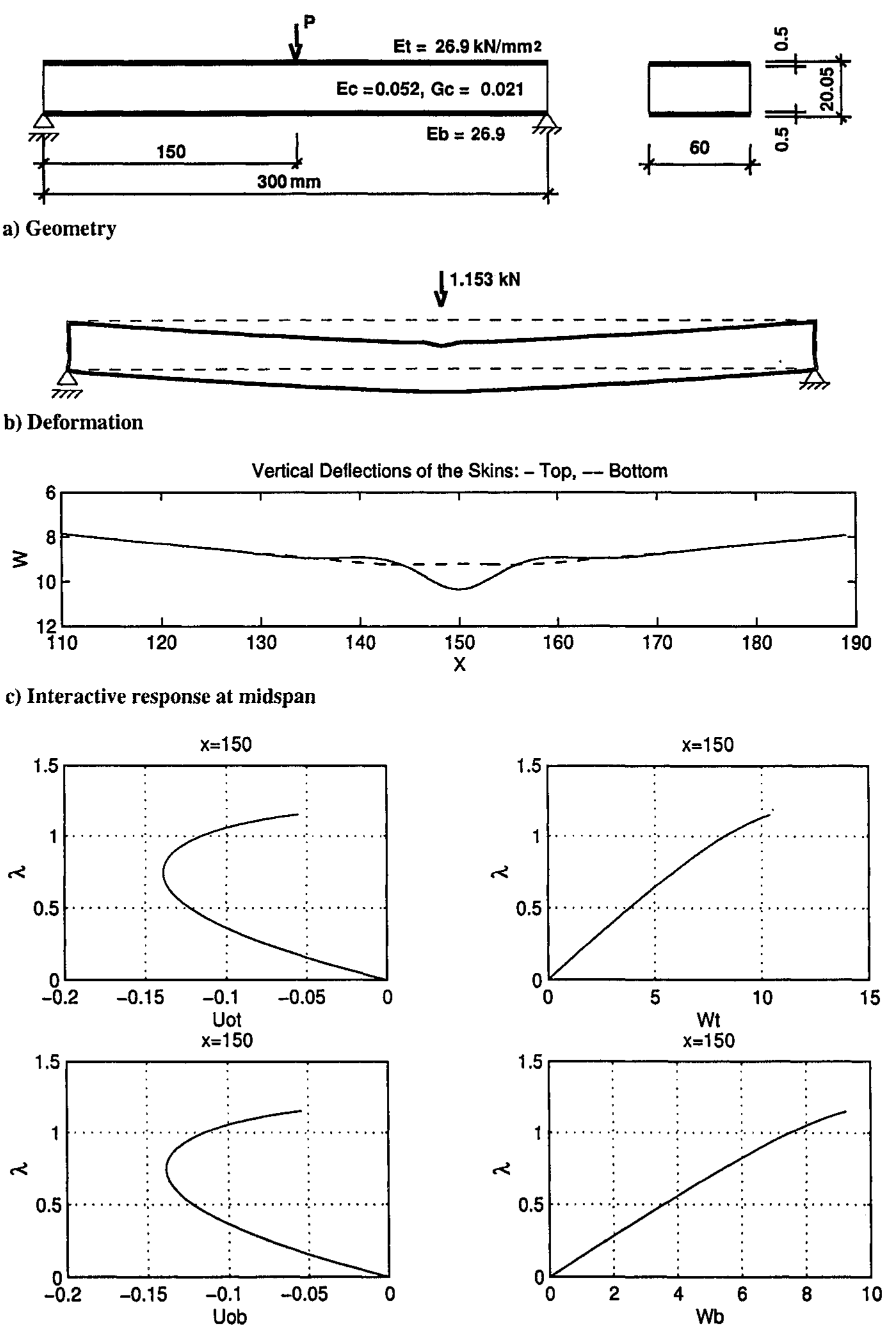


Fig. 3 Sandwich panel of symmetric section under concentrated load at midspan.

of the unknowns for the stress and the deformation fields of the transversely flexible core reduces the two-dimensional formulation to a one-dimensional one.

III. Nonlinear Numerical Analysis

The nonlinear governing differential equations (1–5), the corresponding boundary conditions (8–11), and continuity conditions (12–19) can be written as

$$L(\boldsymbol{v}) = \boldsymbol{F} \tag{25}$$

where $\boldsymbol{L}$ is the nonlinear differential operator, $\boldsymbol{v}$ is the vector function of the unknowns, and $\boldsymbol{F}$ is the vector function of the external loading.

The exact continuous formulation of Eq. (25) is approximated by finite differences and modified to produce

$$G(\boldsymbol{u}, \lambda) = \boldsymbol{0} \tag{26}$$

where $\boldsymbol{G}$ is the nonlinear algebraic operator from $\mathbb{R}^n \times \mathbb{R}$ to $\mathbb{R}^n$ in a finite dimensional space of size n; $\boldsymbol{u}$ is the n-dimensional vector of the unknowns; and λ is a load-level parameter that multiplies a fixed vector function of external loading $\boldsymbol{G}_\lambda$ to yield $\boldsymbol{F}$.

The emphasis is made that the choice of the type of discretization has been inspired by the lack of a finite element of sandwich panel with a soft core in the library of modern finite element packages and the presence of the highly stressed zones near the concentrated loads and in the vicinity of the support points. The complicated response at these regions is smeared by the finite elements.[10]

The system of nonlinear algebraic equations in Eq. (26) defines the equilibrium state of the sandwich panel. The equilibrium state is the point in $(\boldsymbol{u}, \lambda)$ space, whereas the equilibrium branch is a connected curve consisting of such points. Evaluation of the nonlinear equilibrium path for the sandwich panel with the soft core is the main goal of the present work. Considering the mixed-form formulation of the problem on the one hand and the chosen approximation technique on the other, to apply the more general continuation procedures than the ones given in Refs. 11 and 12 was found to be advantageous.

Choosing the load factor λ as the parameter of the equilibrium branch leads to the natural parameter continuation procedure for computing the solution curves of Eq. (26). This procedure, however, fails to trace the curve in the vicinity of the limit (turning) points. To permit the solution of such situation, the natural parameter continuation must be abandoned in favor of some other parameterization. The present paper uses the scalar normalization proposed by Keller[13,14]:

$$N(\boldsymbol{u}, \lambda, \Delta s) \equiv \frac{\mathrm{d}\boldsymbol{u}_0^T}{\mathrm{d}s}(\boldsymbol{u} - \boldsymbol{u}_0) + \frac{\mathrm{d}\lambda_0}{\mathrm{d}s}(\lambda - \lambda_0) - \Delta s = 0 \tag{27}$$

where $(\boldsymbol{u}_0, \lambda_0)$ is any point on the equilibrium path; $(\mathrm{d}\boldsymbol{u}_0^T/\mathrm{d}s, \mathrm{d}\lambda_0/\mathrm{d}s)$ is the unit tangent to the equilibrium path at the point $(\boldsymbol{u}_0, \lambda_0)$; Δs is a distance from the point $(\boldsymbol{u}_0, \lambda_0)$; and symbol T denotes vector transpose. Combining Eqs. (26) and (27) produces

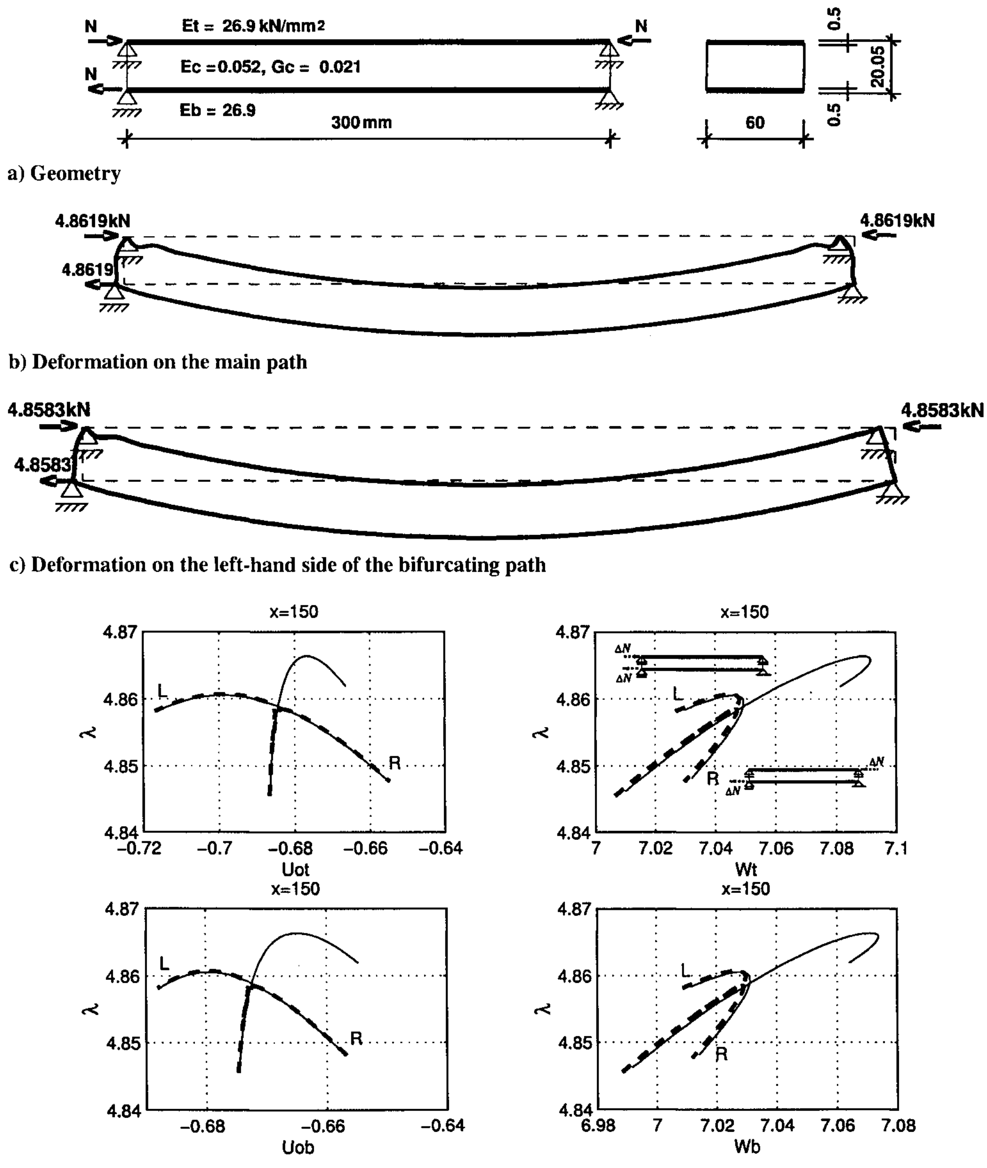


Fig. 4 Sandwich panel of symmetric section and free core edges under end couples: - - -, imperfection analysis.

an extended system of $n+1$ scalar equations for the $n+1$ unknowns $(\boldsymbol{u}, \lambda)$:

$$\begin{bmatrix} G(\boldsymbol{u}, \lambda) \\ N(\boldsymbol{u}, \lambda, \Delta s) \end{bmatrix} = 0 \tag{28}$$

Computing the solution curve with the aid of the last system is known as the pseudo-arc-length continuation procedure.[14] The basic principles of organization of an automatic path-following procedure can be found, for example, in Refs. 11–15.

Starting from the unloaded state of the structure, the continuation procedure advances from one equilibrium state to the other using the predictor-corrector technique. The path-following procedure in the present paper implements the Euler tangential predictor and the quasi-Newtonian global framework with line searches[16] to find the solution of Eq. (26) as well as of the bordered equations, Eq. (28). Moreover, the automatic step-length control that combines sufficiently fast advance along the curve with reliable location of branching points (bifurcation and limit) has been developed.

The change of the determinant sign during continuation along the equilibrium path is used as the branching test function.[14,15] After the branching point is straddled, its value is determined by the bisection procedure and then further refined with the aid of the interpolation suggested in Ref. 15. The test for a limit point or bifurcation at each singularity reads[13]

$$\psi^T G_\lambda \begin{cases} \approx 0 & \text{for bifurcation} \\ \neq 0 & \text{for a limit point} \end{cases} \tag{29}$$

where ψ is the left null vector of the Jacobian G_u at the singular point, whereas G_λ is the fixed vector of the external loading.

Switching branches at bifurcation points is performed, in the present work, using the method proposed by Keller.[13,14] The main idea behind this method is as follows: If one branch through the bifurcation point is already computed, the tangent $(\mathrm{d}\boldsymbol{u}^T/\mathrm{d}s, \mathrm{d}\lambda/\mathrm{d}s)$ at the bifurcation point on this branch is known. The solution on the emanating branch is sought with the aid of the vector, which is "orthogonal" to the known tangent and lies in the plane spanned by vectors $(\phi, 0)$ and $(\phi_0, 1)$, where ϕ is the right null vector of the Jacobian at the singular point G_u^0, and vector ϕ_0 is defined by the system

$$G_u^0 \phi_0 = -G_\lambda, \qquad \psi^T \phi_0 = 0 \tag{30}$$

Reversing the direction of the constructed orthogonal vector, the other part of the bifurcating branch is obtained.

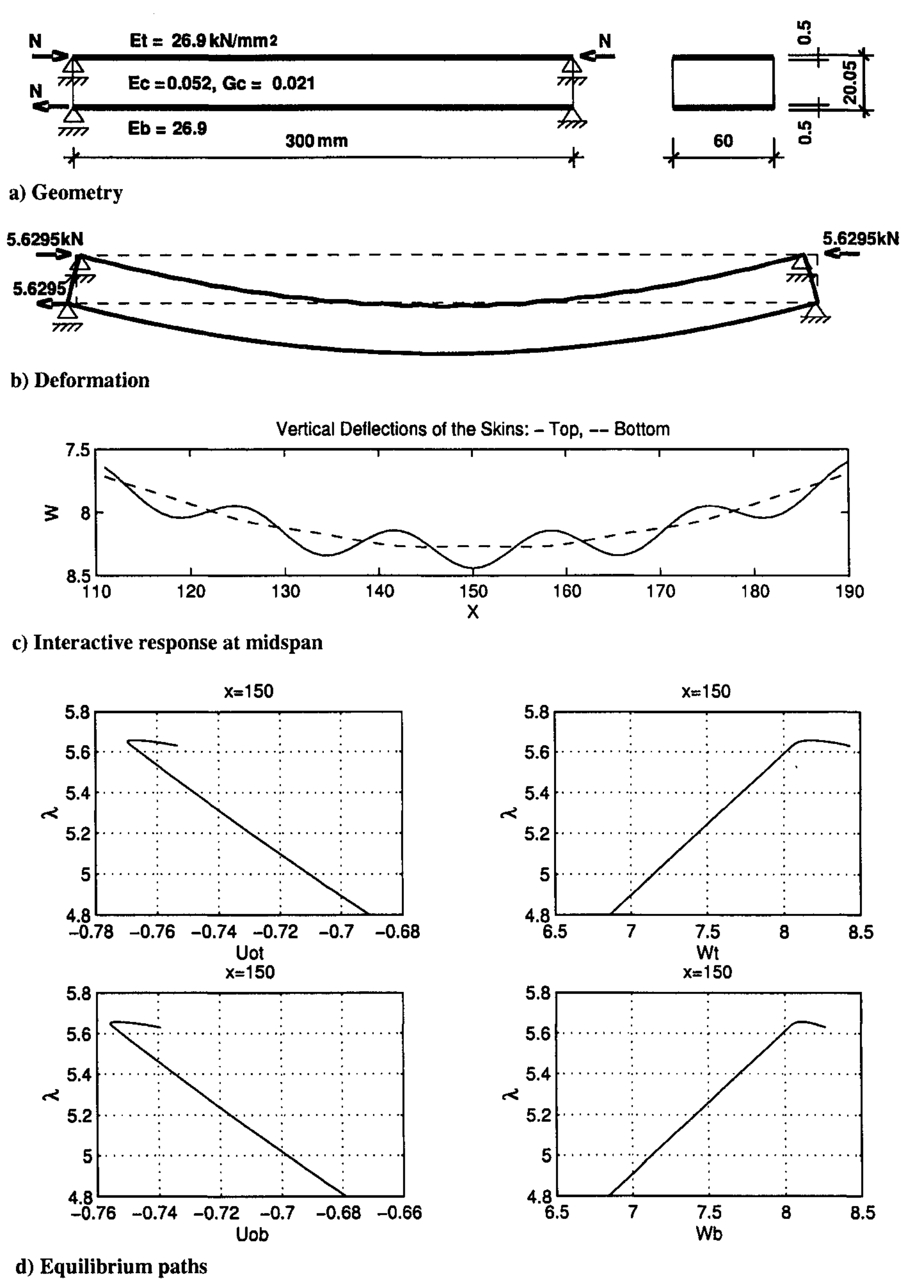


Fig. 5 Sandwich panel of symmetric section and attached rigid plates under end couples.

The equilibrium states, which are very close to the bifurcating branches, can be also determined using the imperfection analysis (Ref. 9; in the mathematical literature it is also known as the "perturbed bifurcation" method[14]). Giving the structure a small initial imperfection, the equilibrium branch can be evaluated by passing the bifurcation point. The smaller the initial imperfection, the closer the "perturbed" equilibrium path to the bifurcating branch.

The nonlinear procedure just outlined has been implemented in the program FSAN developed in the MATLAB® software environment.[17] The numerical study of the following section has been conducted with the aid of this program.

IV. Numerical Study and Discussion

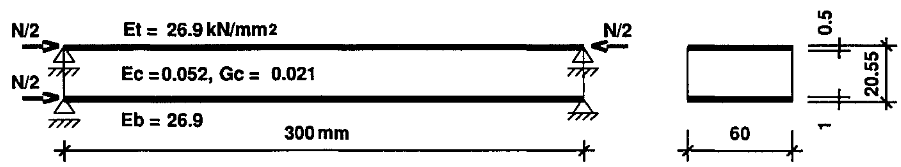
The nonlinear analysis described is applied to various sandwich panel configurations that have been investigated previously for linear bending[1] and buckling[2,8] behavior with symmetric and asymmetric layers. The asymmetric configuration chosen consists of two nonidentical face sheets where the lower face sheet is twice the thickness of the upper one and height of the soft core is kept unchanged. In what follows three qualitatively different types of external loading have been applied to typical sandwich panel configurations.

A. Example 1: Symmetric Section—Concentrated Load at Midspan

The first example deals with a sandwich panel of symmetric section that is loaded by the line load at its midspan (Fig. 3a). The panel is supported only at the lower face sheet, namely, by the roller at the left end and the pinned support at the right end. The nonlinear response of the panel appears in Fig. 3b. The deformation patterns in the vicinity of the load and the supports are the results of the interaction between the overall and the localized modes as shown in Figs. 3b and 3c. The amplitudes of the localized modes and their extension along the span of the panel grow gradually with increase in the load factor λ. The nonlinear equilibrium paths in Fig. 3d reveal that the vertical deflections of the panel remain linear for a wide range of variation of the external loading, whereas the horizontal displacements exhibit strong nonlinearity from the very outset. The path-following procedure here is terminated when the convergence criteria of an overall equilibrium and a slope for the class of deformations considered are exceeded.

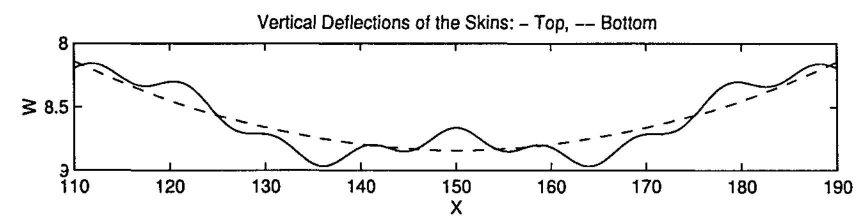
B. Example 2: Symmetric Section—End Couples

The next example considers a sandwich panel with a symmetric section subjected to the action of end couples (Fig. 4a). The couples

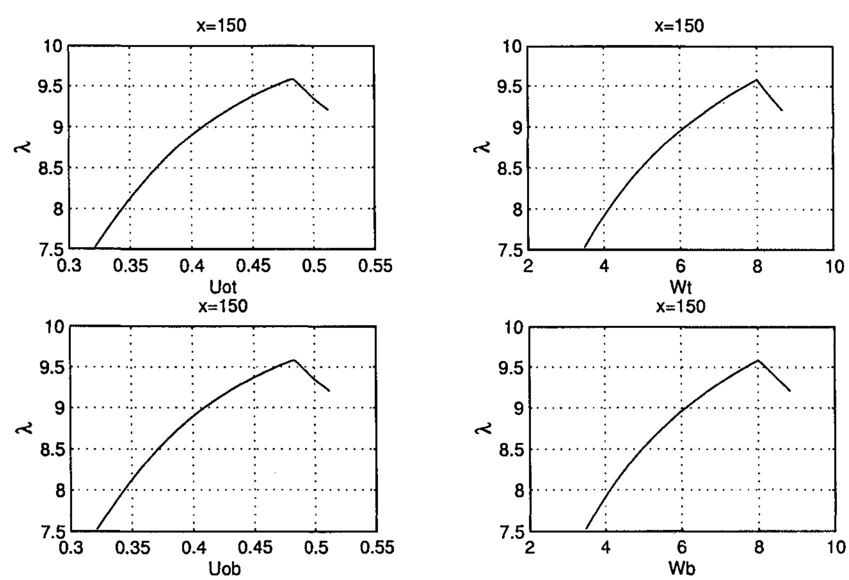


a) Geometry

b) Deformation



c) Interactive response at midspan



d) Equilibrium paths

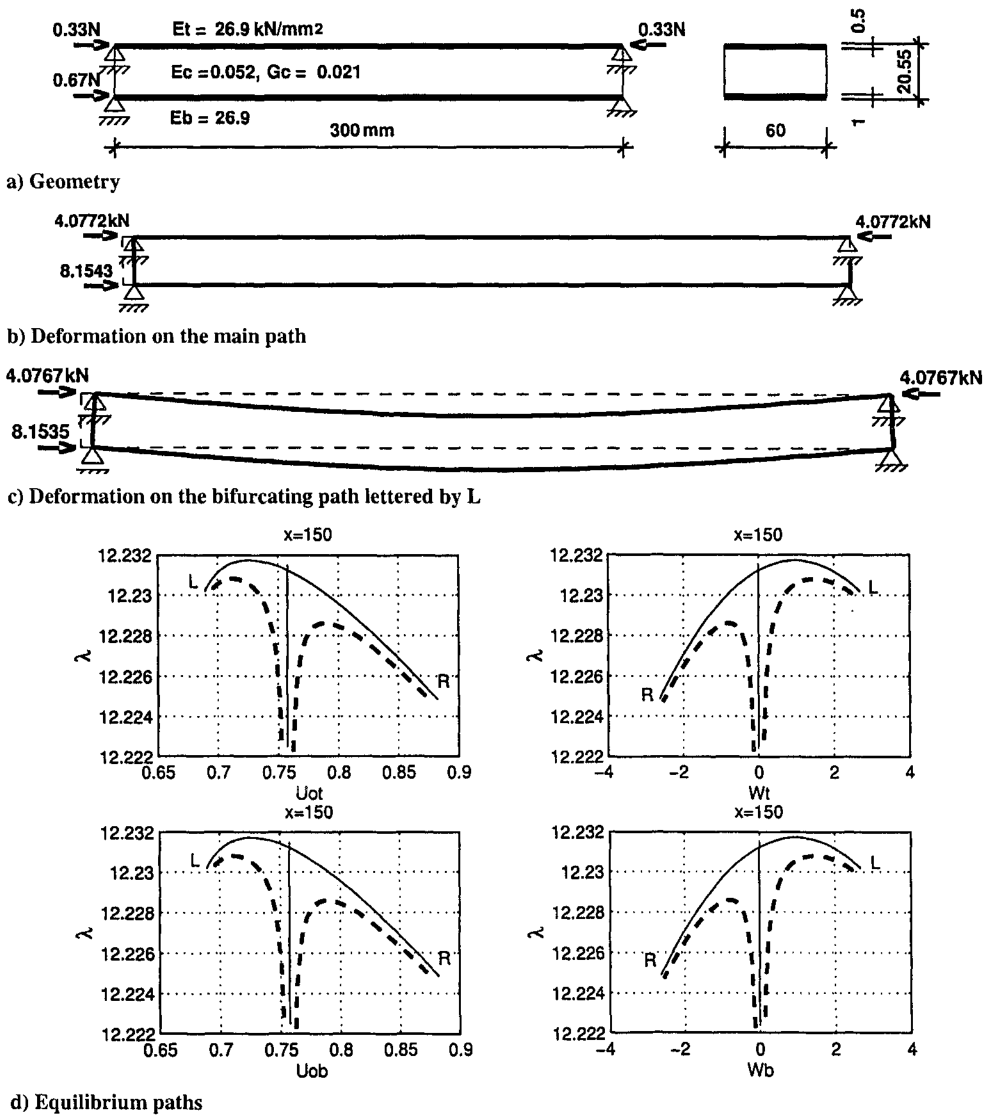
Fig. 6 Sandwich panel of asymmetric section under compressive longitudinal forces equally distributed between the face sheets.

consist of the longitudinal forces exerted on the face sheets. The upper face sheet of the panel is supported by rollers, and the lower face sheet has a roller at its left edge and a pinned support at its right edge. In addition the edges of the panel are free of shear traction. The continuation procedure has detected the bifurcation point, at $N_{\mathrm{b.p.}} = 4.8589$ kN, and the limit point, at $N_{\mathrm{l.p.}} = 4.8664$ kN, on the main equilibrium branch (the ascending solid line) (Fig. 4d). The interactive nonlinear response along this branch is the combination of the overall mode and the localized deformation patterns in the vicinity of the support zones (Fig. 4b). The emanating branch (the solid line crossing the ascending main equilibrium branch in Fig. 4d) is obtained using the method described in Sec. III. The left-hand part of this branch (denoted by letter L in Fig. 4d) also possesses a limit point at $N_{\mathrm{l.p.}} = 4.8606$ kN. The interactive nonlinear response along this branch appears in Fig. 4c. It is asymmetric in the vertical direction as opposed to the symmetric vertical displacements pattern along the main equilibrium path (compare Figs. 4b and 4c). The right-hand, descending, part of the emanating branch (denoted by letter R in Fig. 4d) exhibits the nonlinear behavior that is a mirror image of the left-hand part of the bifurcating branch. Therefore, the symmetry-breaking asymmetric postbuckling response occurs in this case, and the bifurcation point itself is unstable.[12] Moreover, because the sensitivity to the imperfection is completely determined by the character of the equilibrium state at the bifurcation point,[9] the sandwich panel considered is an imperfection-sensitive structure.

The same conclusion can be also obtained using the imperfection analysis (see Sec. III). The initial imperfections are simulated by the small fictitious end couples identical to those of the actual loading but remaining unchanged during the loading process. These fictitious couples are applied to the left and the right edges of the panel to produce the asymmetric initial imperfections in the sandwich panel. Thus, to determine the left dashed line in Fig. 4d, the left fictitious end couple is applied in addition to the external loading shown in Fig. 4a. This fictitious couple is produced by two small longitudinal forces ΔN acting at the left edge of the panel in the directions of the actual forces. Similarly, the right dashed branch is determined using the right fictitious end couple in addition to the external loads. The right fictitious couple is produced by two small longitudinal forces ΔN acting at the left bottom and the upper right ends of the panel in the directions of the actual external loading (see the sketches in the upper-right-hand plot of Fig. 4d).

The linear buckling analysis of this case yields a critical load that exceeds its nonlinear counterpart by only 1.21%. The corresponding buckling mode is also in a close agreement with the nonlinear result.[8]

A strong qualitative change in the response of the panel just discussed occurs when the vertical displacements of the core at the edges of the panel are prevented (Fig. 5). Physically, such a boundary requirement can be interpreted as the attachment of thin rigid plates to the core and the face sheets at the edges of the panel. Such plates fully prevent the vertical displacements of the core at its edges while permitting the free longitudinal displacements. As can be seen from the branching diagrams of Fig. 5d, the bifurcation point disappears, giving way to the arising of a limit point. Thus, precluding the vertical displacements at the edges of the panel yields a nonlinear



d) Equilibrium paths

Fig. 7 Sandwich panel of asymmetric section under compressive longitudinal forces nonequally distributed between the face sheets: - - -, imperfection analysis.

response that is nonsensitive to small initial imperfections. At the load level of about 5.41 kN, the overall response begins to change into the interactive one, thus leading to the appearance of waves at the compressed upper face sheet near the supports. The waves propagate gradually toward the midspan of the panel with increase in the load factor. At the limit point $N_{\text{l.p.}} = 5.657$ kN, the upper face sheet develops a wrinkle appearance; passing the limit point yields waves that tend to grow more intensively at the midspan rather than at the support zones. The linear buckling analysis in this case was incapable of evaluating a real eigenvalue.

C. Example 3: Asymmetric Section—Compression by Longitudinal Forces

In this example a sandwich panel with an asymmetric section and subjected to longitudinal compression is considered. Here the thickness of the lower face sheet is twice that of the upper one, and the vertical deformations at the edges of the panel are prevented (Fig. 6a). The nonlinear equilibrium path possesses a limit point at $N_{\text{l.p.}} = 9.581$ kN (Fig. 6d). The nonlinear behavior of the panel along this path experiences the qualitative change from the starting overall deformation pattern to the interaction of the overall mode with the local one that consists of the wrinkling of the upper face sheet (Figs. 6b and 6c). The equilibrium state at the limit point is known to be unstable,[12] thus the panel exhibits a snap-through nonlinear response similar to the results for the panel with the attached rigid plates and subjected to the end couples. However (Figs. 6c and 5c), the deformation patterns of the face sheets in these two cases are different.

A comparison between the linear buckling results and the nonlinear analysis reveals that the linear buckling analysis yields a bifurcation load at $N_{\text{b.p.}} = 11.394$ kN, whereas the panel in fact exhibits a limit-point behavior with a load of 16% lower than the linear bifurcation load; the buckling mode, however, is qualitatively correct.

In the next case the compressive forces applied to the preceding asymmetric layout are divided proportionally to the stiffnesses of the face sheets yielding an initial membrane state (Fig. 7a). The response diagram in Fig. 7d presents a transcritical bifurcation[15] with an unstable bifurcation point $N_{\text{b.p.}} = 12.231$ kN. The dashed lines in Fig. 7d, as in the preceding example, are the results of an imperfection analysis. The perturbed solutions (see Sec. III) have been determined by simulating the initial imperfections with the aid of small vertical external loads q_t and q_b, uniformly distributed along the span of the panel [see Eqs. (1–5)] and kept unchanged during the loading process. The redistribution of the external longitudinal loading between the face sheets caused a qualitative shift in the panel response from an imperfection-nonsensitive to an imperfection-sensitive one. Here the buckling results are in complete agreement with the linear buckling analysis.

The examples discussed reveal some important features of the nonlinear sandwich panel behavior. The interactive nonlinear response manifests itself as an interaction of the overall (global) mode with the localized modes (example 1 and the first case of example 2) as well as local modes (the second case of example 2 and the first case of example 3). As can be seen from Figs. 5 and 6, the initial overall mode that corresponds to low load levels gradually transforms into a nonlinear interactive mode response as λ increases. Such a transformation exhibits a gradual change in the panel behavior without yielding buckling of the panel as a whole. Note, however, that the progress along the main path does not affect the general nature of the deformation pattern as well as the state of stresses. The qualitative changes in the response take place when going from the main equilibrium path to the bifurcating branch (Figs. 4 and 7). The peculiarities of the nonlinear phenomena determined are caused by the interaction of the face sheets with the soft core during loading. They cannot be obtained without accounting for the transverse flexibility of the core or by using a decoupling of modes type of analysis.

V. Conclusions

The geometrically nonlinear analysis based on the closed-form HSAPT[2] is applied to study the response of the sandwich panels with the transversely flexible core in the range of intermediate class of deformations. The theory enables a general solution approach that does not resort to decoupling of the complicated sandwich response into presumed isolated modes and permits the imposition of different support conditions through the height of the same section. The panels under investigation consist of symmetric and asymmetric sections with the same height of the transversely flexible core. Three distinct loading cases are considered: a concentrated line load at midspan, end couples in connection with a symmetric layout, and compression of the asymmetric panel by longitudinal loads.

The pseudo-arc-length continuation procedure, based on the quasi-Newtonian global framework with line searches as the solver, is described and used to compute the equilibrium paths for the sandwich configurations under consideration. The algorithm enables one to straddle and evaluate the branch points (bifurcation and limit points) as well as to trace the emanating (bifurcating) branches starting from the already evaluated bifurcation points. The path-following procedure presented performs well and can be used to predict the nonlinear response of the sandwich panels with a transversely flexible core for practical needs.

The numerical investigation reveals that the sandwich panel, as a compound structure, exhibits complicated nonlinear interactive mode response along its equilibrium paths including bifurcations and limit points. The nonlinear interactive mode response appears as an interaction between the overall (global) and the local or localized modes along the span of the panel. The special features of this response are caused by the collaboration of the face sheets with the transversely flexible core during the deformation process. The nonlinear analysis indicates that the interactive mode response is strongly unstable. This finding may propose an introduction of some knock-down factors to the external loads in the design of sandwich panels with the transversely flexible core.

The triggering overall mode may gradually transform into an interactive mode pattern along the same equilibrium branch. In other words, the wrinkling of the compressed face sheet caused by bending does not necessarily imply that buckling of the sandwich panel as a whole has occurred. At the same time the nonlinear behavior along the bifurcating branch exhibits a drastic change in the general nature of the deformation pattern or in the state of stresses when compared to the main equilibrium path, such as the change from symmetry to asymmetry or from the membrane initial state to bending. Furthermore, the observation has been made that the loading cases with interactive modes which consist of small contributions from the overall mode can be analyzed for critical buckling behavior using the linearized approach with minor inaccuracy.

Sandwich panels are very sensitive to variations in the boundary conditions and in the distribution of the longitudinal edge loading, which may change the response of the same panel from an imperfection-nonsensitive to an imperfection-sensitive one.

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